Stochastic Volatility Model For Indian Security Indices: VaR Estimation and Backtesting

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INTRODUCTION

Basel Committee on Banking Supervision, (1996, 2006) prescribes periodic estimation of the Value at Risk (VaR) (Morgan, 1996) to be carried out by asset management companies, banking organisations and similar financial institutions as a mandatory requirement. Many Indian financial institutions currently employ EWMA based method for VaR estimation (Varma, 1999). Das, Basu, Das (2008) applied the EWMA, GARCH, EGARCH and GJR-GARCH for volatility estimation towards VaR prediction using recent data from Indian Security Market and empirically shows that EGARCH outperforms over others. This paper employs data of share index from Indian stock exchange and explores the relative advantages of stochastic volatility model for VaR estimation, keeping in mind the widely held (Shephard, 2005) concept that the quality of volatility estimation not only depends on the mathematical technique used but could also be dependent on country, asset type (specific stock, portfolio, commodity, derivatives etc.) and other factors. Empirical characterization of techniques from actual data is therefore considered as a useful activity. Other recent examples of using country-specific market data are Hagen and Yu (2001), Yu (2002), Ekrem (2004), Fuh and Yang (2007).

Stochastic Volatility Model (SVM), possibly first suggested in Nelson (1988), is a distinctly different approach from EWMA and ARCH/GARCH models. The relative advantages of SVM approach (as recently reviewed by Broto and Ruiz (2004) and Shephard (2005) include capability to provide one-step-ahead prediction and to better accommodate excess kurtosis and leverage effects compared to GARCH. The disadvantage of SVM [ibid] includes the requirement of simultaneous estimation of states and parameters, and consequent additional computation.

The applicable numerical techniques for this simultaneous estimation of states and parameters as surveyed in Broto and Ruiz (2004) and Shephard (2005) include the moment matching method (Shephard, 2005), Quasi-Maximum Likelihood (QML) (Breidt and Carriquiry, 1996), and Monte Carlo simulation based method [Jacquier, Polson, and Rossi, 1994). More elaborate discussions on the QML method may be obtained in Harvey, Ruiz and Shephard (1994), Jacquier, Polson and Rossi (1994) and Breidt and Carriquiry (1996). Estimation of stochastic volatility model parameters using QML technique is usually based on Kalman Filter (KF) (Brown and Hwang, 1997). Kalman Smoothing for stochastic volatility estimation in a state space model for real and simulated situations are discussed in Jacquier, Polson and Rossi (1994).

Objectives of the research reported herein are two folds: firstly, to investigate the predictive power of the stochastic volatility model (SVM), for VaR prediction using data from the Indian Market and to detect any country specificity; secondly, to obtain some benchmark results with which one can compare results from newer parameters and state estimation techniques (Broto and Ruiz, 2004) now made feasible due to increased computational power at affordable cost. Towards the above objectives, this paper specifically investigates the efficacy of the SVM for VaR prediction using data from Indian security indices. The QML method has been used for obtaining the parameters of the SVM, whereas the KF has been used for state estimation, leading to the VaR estimation. The VaR estimates are back tested and such results are compared with results obtained through other techniques, namely EWMA and GARCH based approaches. It is to be noted that the efficacy of the model and techniques are compared through backtesting the empirical VaR estimates instead of statistical test of significance.

The rest of the paper is organised as follows. The next section briefly states the steps involved in the QML-KF based SVM analysis and VaR estimation. The third section presents the performance of the above SVM method using data from the Indian stock market and compares the same obtained with EWMA and GARCH approaches. The final section summarizes the conclusions.

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THE SVM APPROACH TO VaR ESTIMATION

Discrete Time Stochastic Volatility Model

The discrete time model of the risky component of the return y_t given in Taylor (1982), is a product process: $y_t = \sigma_t.\varepsilon_t$ (1)

where ε is zero mean unit variance Gaussian process and σ_t is some non-negative process , termed as the "latent stochastic volatility".

The state space stochastic volatility model (Broto and Ruiz, 2004) with the latent stochastic volatility, is characterised by Nelson (1988), the substitution assuming $\sigma_t = \exp(h_t/2)$ whereby $y_t = \varepsilon_t.\exp(h_t/2)$, implying

$$\log y_t^2 = h_t + \log \varepsilon_t^2, \tag{2}$$

$$h_t = \alpha + \delta.h_{t-1} + \sigma_v v_t \text{ for } t = 1,2,........,T \tag{3}$$

The above is a discrete time state evolution process, of which, h_t is the state variable, the log return y_t is the measurable output. The parameters δ, α and σ_v characterize the state evolution, with δ, the state evolution gain (must have a modulus less than unity) for stability, α a constant but unknown bias input, v_t zero mean, unit variance normal Gaussian noise (process noise) sequence and σ_v is a positive constant giving the standard deviation of the process noise. In the measurement equation, $\eta_t = \log \varepsilon_t^2$ is analogous to measurement noise. Though ε_t may be a zero mean, unit variance normal Gaussian sequence, $\eta_t = \log \varepsilon_t^2$ is not. However, from Breidt and Carriquiry (1996), it may be verified that the mean and variance of $\log \varepsilon_t^2$ are -1.27 and $\frac{\pi^2}{2}$ respectively.

The term $\log \varepsilon_t^2$ is often replaced by a zero mean noise η_t,

$$\text{i.e. } \log y_t^2 = -1.27 + h_t + \eta_t,\ E(\eta_t) = 0,\ VaR(\eta_t) = \frac{\pi^2}{2}, v_t \sim N(0,1) \tag{4}$$

The usage of the SVM for VaR computation calls for (i) identification of the parameters of the model, (ii) estimating the latent state variable h_t (iii) using the state equation for extrapolating h_{t+1} and thereby, the log return y_{t+1} (iv) using the above for VaR computation.

SVM PARAMETER ESTIMATION USING QML-KF

The Quasi-Maximum Likelihood (QML) technique is one of the simplest but an approximate method for parameter estimation of the SVM. As the noise process is η_t non Gaussian, standard parameter estimation techniques cannot be used directly. However, the QML method makes the approximation by assuming that η_t is Gaussian so that Kalman Filtering method may be used. Following the Kalman filtering step and numerically determining the point in the parameter space, $\alpha,.\delta,\sigma_v$ corresponding to the maximum likelihood of the measurement sequence { y_t } or equivalently of { $\log y_t$ }. The advantage of following QML approach is its speed and adaptability to diversified situations (Broto and Ruiz, 2004). To initiate the Kalman filter steps, one must have initial distribution (mean and covariance) of the state variable h_t at t=0. It is customary to assume $h_0 \sim N(\alpha/(1-\delta),\ \ \sigma^2{}_v/(1-\delta^2))$, due to stationarity of the underlying time series (Harvey, Ruiz and Shephard, 1994).

ALGORITHM: QML-KF

Step 1: Initialization Step

Set t=0. Select suitable guess values of α, δ, σ_v. Initialise the filter state by

$h_o = \dfrac{\alpha}{1-\delta}$ and state error covariance $\Omega_o = \dfrac{\sigma_v^2}{1-\delta^2}$

Step 2: KF Prediction Step

$h_{t/t-1} = \alpha + \delta . h_{t-1/t-1}$ and

$\Omega_{t/t-1} = \delta^2 . \Omega_{t-1/t-1} + \sigma_v^2$,

Step 3: KF Update Step

$$h_{t/t} = h_{t/t-1} + \frac{\Omega_{t/t-1}}{f_t}\left[\log(y_t^2) + 1.27 - h_{t/t-1}\right] \text{ and}$$

$$\Omega_{t/t} = \Omega_{t/t-1}\left[1 - \frac{\Omega_{t/t-1}}{f_t}\right]$$

where $f_t = \Omega_{t/t-1} + \frac{\pi^2}{2}$

Step 4: Repeat Step2 and 3 for all time steps i.e. for t = 1, 2,….,T.

Step 5: Likelihood Calculation

$L(\alpha,.\delta,.\sigma_v) \propto -\frac{1}{2}\sum \log(f_t) - \frac{1}{2}\sum \frac{e_t^2}{f_t}$ where $e_t = \log(y_t^2) + 1.27 - h_{t/t-1}$ is the one step ahead prediction error.

Step 6: Repeat Step 1 to 5 to calculate $L(\alpha, \delta, \sigma_v)$ for a suitable range covering the expected range of values with appropriate increments.

Step 7: Determine α^*, δ^* and σ_v^* for which $L(\alpha^*, \delta^*, \sigma_v^*)$ is maximum and these α^*, δ^* and σ_v^* are the ML estimates of three parameters under consideration respectively.

VaR ESTIMATION AND BACKTESTING

The steps for VaR (1% confidence level) calculation (Campbell, 2005; Morgan, 2006; Lehikoinen, 2007; Das, Basu, Das, 2008) from volatility (σ_t) are as follows:

Step 1: Calculate VaR for the scrip at 3.5σ level and VaR for the index at 3σ level. (A higher σ level is used for the scrip because scrip is expected to have higher volatility as compared to the index, which is a portfolio. The volatility estimate at 3σ level represents 99% level).

Step 2: Calculate VaR for a security or index for a particular day using the σ for both long position and short positions.

For Scrip

VaR for Short Positions= $Exp(3.5\sigma) - 1$

VaR for Long Positions= $1 - Exp(-3.5\sigma)$

For Index

VaR for Short Positions= $Exp(3\sigma) - 1$

VaR for Long Positions= $1 - Exp(-3\sigma)$

Step 3: To ensure that risk for all possible situations is covered, Long VaR or Short VaR, whichever is higher, is to be considered as the VaR for the scrip or index, as the case may be.

To evaluate the goodness of a VaR model, banks, financial institutions as well as regulators use backtesting to confirm their judgments (Lehikoinen, 2007). Backtesting a VaR model simply means checking whether the realized daily returns are consistent with the corresponding daily VaR produced by the model at the given confidence level (typically 99%). This is carried out by evaluating the difference in the values estimated by the VaR system and the ex-post mark-to-market portfolio value. This backtesting analysis can be interpreted as a static statistical test on the validity of the VaR estimation methodology. If losses in excess of the reported VaR takes place more frequently than (0.01) X100 % of the time, then this would suggest that the reported VaR measure systematically understates the portfolio's actual level of risk. The converse finding of too few VaR violations would alternately signal an overly conservative VaR measure. If N is the number of 1% VaR violations in the previous 250 trading days, then percentage violation is defined by (NX100)/250. The green, yellow and red zone in traffic light framework is defined by $N \leq 4$, $5 \leq N \leq 9$ and $N \geq 10$ (Campbell, 2005; Basel, 1996, 2006). In our implementation, we have back tested the VaR estimates for the last 250 days and the results are given in Table 2 and 3.

EMPIRICAL INVESTIGATIONS

Data Selection

The implementation is carried out with the closing values of nine NSE indices including S & P CNX NIFTY and SENSEX daily closing values observed during the time period– 2nd January 2006 to 5th November 2007 (total observation of 460 days). 10 years Sensex and Nifty data, from 2nd February 1998 to 31st January 2008, have also been chosen for parameter estimation (total observation of 2479 days for Sensex and 2512 for Nifty).

RESULTS AND DISCUSSIONS

Using the algorithm described in section (2.2), all the 10 indices are analyzed to estimate the model parameters listed in two major Indian Security Markets. The QML estimates of three parameters namely $\hat{\alpha}$, $\hat{\delta}$, $\hat{\sigma}_v$ are given in Table 1. Subsequently, the technique illustrated in section (2.2) are used to estimate the latent volatility (σ_t) for the 10 Indian stock indices during the said period for which $\hat{\alpha}$, $\hat{\delta}$, $\hat{\sigma}_v$ took place. The estimated volatilities are used for VaR estimation and backtesting sketched in section (2.3).

Table 1: QML Estimate of Stochastic Volatility Model Parameters

Name of Index	$\hat{\alpha}$	$\hat{\delta}$	$\hat{\sigma}_v$
BANK NIFTY	-0.66	0.918	0.30
CNX 100	-0.46	0.947	0.34
CNX IT	-0.51	0.940	0.29
CNX MIDCAP	-0.82	0.906	0.37
CNX NIFTY JUNIOR	-0.6	0.929	0.34
NIFTY MIDCAP 50	-1.14	0.865	0.49
S & P CNX 500	-0.48	0.945	0.32
S & P CNX DEFTY	-0.65	0.924	0.44
S & P CNX NIFTY	-0.71	0.919	0.44
SENSEX	-0.45	0.948	0.29
S&P CNX NIFTY 10 Years	-0.88	0.898	0.44
SENSEX 10 Years	-1.25	0.855	0.52

Range & Constancy of Parameters: Estimate of α, δ, and σ_v varies between -1.25 to-0.45, 0.855 to 0.947 and 0.29 to 0.52 respectively which are not too far from those estimated from 500 simulations in Jacquier, Polson and Rossi (1994). From Table 1, it may be noted that across the different indices, the variation of $\hat{\delta}$ and $\hat{\sigma}_v$ are reasonable whereas large variation (1:3) is noticeable for $\hat{\alpha}$. The values of $\hat{\delta}$ is close to that obtained by Jacquier, Polson and Rossi 1994 for US S& P 500.

Table 2: Computing Percentage Violations Using Different Methods (Backtesting last one year (250 days) return with Gaussian residuals)

Name of Indices	EWMA	GARCH (1,1)	EGARCH (1,1)	SVM-QML- KF
BANK NIFTY	5.2	2.4	2.8	3.6
CNX 100	4.8	2.8	3.2	3.6
CNX IT	3.6	2.4	1.6	2.4
CNX MIDCAP	4.8	3.2	3.6	4.4
CNX NIFTY JUNIOR	5.2	2.8	3.6	4.8
NIFTY MIDCAP 50	4.4	3.2	3.2	4.4
S & P CNX 500	4.8	3.2	3.2	3.6
S&P CNX DEFTY	4.8	3.2	3.2	4.4
S&P CNX NIFTY	4.8	2.8	2.8	4.8
SENSEX	3.2	2.8	3.2	1.6

Efficacy of VaR Estimation: Table 2 shows and compares the VaR estimation efficacy of four techniques namely EWMA, GARCH, EGARCH and SVM-QML-KF for different market indices. It seems SVM-QML-KF method outperforms the EWMA method which is the recommended method for Indian Exchanges (Varma, 1999). However, the violation wise GARCH/EGARCH are generally superior (except for the Sensex) to the SVM-QML-KF method.

CROSS-PARAMETER VaR ESTIMATION

Table 3: Computing Percentage Violations Using Cross Parameters SVM-QML-KF [Backtesting last one year (250 days) return with Gaussian residuals]

Name of Indices	Same Duration Sensex Data	500 Simulation	10 years Nifty Data	10 Years Sensex Data
BANK NIFTY	4.4	3.6	4.0	4.4
CNX 100	3.6	3.6	4.4	4.8
CNX IT	2.8	2.4	3.6	4.0
CNX MIDCAP	4.0	4.0	4.8	4.8
CNX NIFTY JUNIOR	4.8	4.0	4.4	4.4
NIFTY MIDCAP 50	4.0	3.6	4.8	4.4
S&P CNX 500	3.6	3.2	4.0	4.0
S & P CNX DEFTY	4.0	4.0	5.2	4.4
S&P CNX NIFTY	4.8	4.0	4.8	5.2
SENSEX	1.6	0.8	1.6	2

Table 3 presents the percentage violations of VaR estimates by SVM with parameters estimated with data from different sources given as column headings. The results show that cross parameter VaR estimates do not help much except for the Sensex. This is indicated by the percentage violations in the green zone for column 2 and 4 and near to that zone for Sensex in a traffic light approach.

CONCLUSION

Stochastic volatility model for VaR prediction has been applied to major share indices in the Indian Market. The Quasi-Maximum Likelihood method with embedded Kalman Filtering has been used to identify the model parameters. Though the QML-KF is an approximate method, the predictive power of the method is quite close to that obtained from established methods like EWMA, GARCH (1, 1) and EGARCH, as evident from the back testing performance. Granted that EGARCH or even GARCH(1,1) results are noticeably superior to that obtained from QML-KF, the QML-KF results are better than that obtained from EWMA, for VaR prediction (but marginally so, as NIFTY and Sensex provide contradictory performance). EGARCH, as in our earlier studies, remain the overall best performer. This provides an impetus to carry out further work using the SV method.

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now see India as an "investment destination". Real estate and equity markets are the principal areas of their interest. These sectors, restricted to NRIs in the past, are experiencing a boom. Real estate experts believe that in Delhi, 20 percent of all properties worth over one crore were bought or funded by NRIs. Even second generation Indians are buying property in India. The new found interest in the real estate and equity markets is another explanation for increase in local withdrawals in RBI's remittances figures. NRIs may have finally become "investors" rather than saver.

CONCLUSION

India has clearly achieved a large sustained level of remittances. Policy initiatives by the government and banking institutions have achieved significant results. First, most remittances flow through formal channels. Second, an increasing number of remitters have moved from pure savers to investors. The Indian policy regime has demonstrated its ability to attract NRI capital through NRI deposit accounts and successive bond issues. The challenge is to channel some of these flows for socio-economic development. If the government and the banking community are strategic, they could offer higher rates of return on remittance receipts placed in specified assets in the domestic capital market. Investing in microfinance operations would be a good place to start, given their success in India. The government could issue bonds targeted for infrastructure development or for investments in health and education sectors. The Indian diaspora has proven responsive to incentives. Offering investment options that are tied to development goals could be a winning strategy.

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Use of multitude of market indices and parameter estimation for SVM also provided some glimpse of insight into the Indian security market. Sensex and NIFTY being the two leading (and competing) indices with substantially dominating capitalization, one would expect that the SVM parameters would be comparable. The results obtained do not support this assumption. In fact, the bias parameter α for the two indices showed wide variation. Though it is generally recommended to use about 2 to 5 years market data to obtain the SVM parameters, the same for 2 years and 10 years in case of Sensex were found to be noticeably different. In this respect, NIFTY parameters for 2 years and 10 years have been rather consistent. Variations of parameters across the indices generally discourage application of parameters for one index to be applied for VaR prediction of the others. However, such an exercise did not create any catastrophically wrong prediction as indicated in Table 3. This may be an indicator that the VaR predictor may not be very sensitive to the bias parameter α. Work is continuing to analyse whether the SVM approach brings out significant information regarding country and index specificity.

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